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An overview of Electrostatics

Coulomb's law

Coulomb's law state that the force F between two point charges Q and q is:-

i/ Along the line joining them

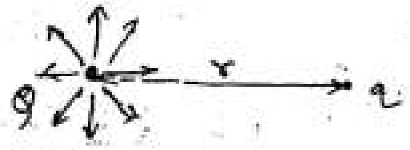
ii/ Directly proportional to the product of charges.

iii/ Inversely proportional to the square of the distance r between them.

Expressed mathematically,

$$F \propto Qq$$

$$F \propto \frac{1}{r^2}$$



i.e., $F = K \cdot \frac{Qq}{r^2}$

where K is the proportional constant.

$$K = \frac{1}{4\pi\epsilon}$$

$\epsilon = \epsilon_r \epsilon_0$

ϵ = permittivity of the medium.

ϵ_0 = Premittivity of the air, or free space. $\epsilon_r = 1$ for air.

value of $\epsilon_0 = 8.854 \times 10^{-12} = \frac{10^{-9}}{36\pi}$ F/m.

$$\boxed{\therefore F = \frac{Qq}{4\pi\epsilon_0 r^2}}$$

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If point charges Q and q are located at points having vectors r_1 and r_2, then force.

$$\boxed{\vec{F} = \frac{Qq}{4\pi\epsilon_0 r^2} a_r}$$ where, $r = |r_1 - r_2|$

Electric field

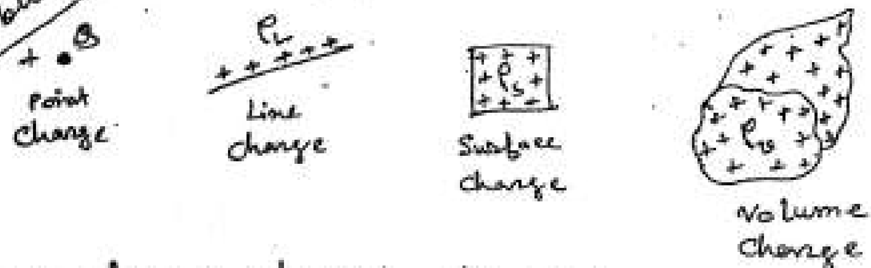
The electric field intensity (or electric field strength) E is the force per unit charge when placed in ~~an~~ ~~electric field~~ an electric field.

$\therefore \vec{E} = \frac{\vec{F}}{q}$ due to a point charge

$$\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\epsilon_0 r^2} a_r$$

[Note, the direction of electric field is equal to the direction of ...]

Electric field due to continuous charge distribution



→ charge element dQ and total charge Q.

→ $dQ = \rho_L dl \rightarrow Q = \int_L \rho_L \cdot dl$ (line charge)

→ $dQ = \rho_s ds \rightarrow Q = \int_s \rho_s ds$ (surface charge)

→ $dQ = \rho_v dv \rightarrow Q = \int_v \rho_v dv$ (volume charge)

1/ Electric field of an infinite long line charge, of charge density ρ_L C/m situated ~~on~~ at z axis. The total electric field having position vector r is

$$E = \int_L \frac{\rho_L dl}{4\pi\epsilon_0 r^2} \cdot a_r \quad \text{(line charge)}$$

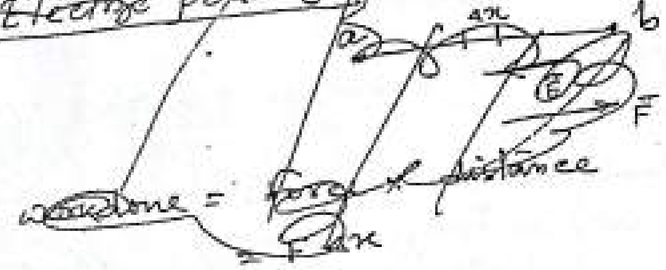
2/ A sheet charge is distributed over the entire xy plane of uniform charge density ρ_s c/m². The electric field having position vector r due to this infinite sheet charge is

$$\vec{E} = \int_s \frac{\rho_s\, ds}{4\pi\epsilon_0 r^2}\, a_r \quad \text{(surface charge)}$$

3/ A volume charge is distributed throughout a sphere of radius a and centered at the origin with uniform density ρ_v c/m³. The total electric field is

$$E = \int_v \frac{\rho_v\, dv}{4\pi\epsilon_0 r^2}\, a_r \quad \text{(volume charge)}$$

Electric potential



Electric flux density ($\vec{D}$)

→ The electric field intensity is dependent on the medium in which the charge is placed.

→ Suppose a new vector field D is defined by

$$\boxed{D = \epsilon_0 E}$$

$$D = \epsilon_0 E = \frac{Q}{4\pi r^2} \text{ coulomb/m}^2$$

From the Gauss's law, $\varphi = \psi$, ψ is total electric flux passing through any closed surface.

$$\therefore D = \frac{Q}{4\pi r^2} = \frac{\psi}{4\pi r^2}$$

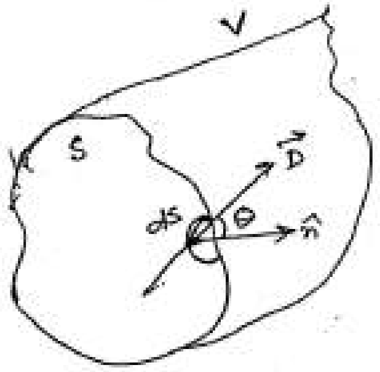
Q is the total charge en. by that surface.

→ The flux due to the electric field E can be calculated by general definition of flux.

$$\psi = \int_S D \cdot dS$$

→ Therefore, the electric flux is measured in Coulombs. Hence, the vector field D is called the electric flux density and unit of D is Coulombs/m^2.
The electric flux density is also known as electric displacement.

Gauss's law



Statement 1

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

$$\psi = \oint_S \vec{D} \cdot \vec{dS} = Q$$

Statement 2

The surface integral of the normal component of the electric flux density or electric displacement $\vec{D}$ over a closed surface is equal to the total change enclosed by that surface.

$$\oint_S \vec{D} \cdot dS = Q$$

If ρ_v is the charge density, then

$$Q = \int_v \rho \cdot dv$$

So, $\oint_s \vec{D} \cdot \vec{ds} = \int_v \rho \cdot dv$

applying Gauss's divergence theorem lefthand ~~both~~ sides of the equation.

$$\iiint \nabla \cdot \vec{D} \, dv = \int_v \rho \, dv$$

$$\boxed{\nabla \cdot \vec{D} = \rho}$$

Statement 3

At every point in a medium the divergence of electric flux density $\vec{D}$ is equal to the volume charge density ρ.

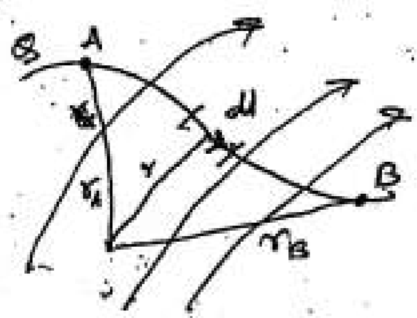
$$\boxed{\nabla \cdot \vec{D} = \rho}$$

Electric potential

→ We can obtain the electric field intensity E due to charge distribution from Coulomb's law in general or, when the charge distribution is symmetric, from Gauss's law.

→ Another way of obtaining E is from the electric scalar potential V. In a sense, this way of finding E is easier because it easier to handle scalar's than vectors.

→ Suppose we wish to ~~re~~move a point charge Q from point A to point B in an electric field E as shown in Fig.



From coulomb's law, the force on Q is

$$F = QE$$

So that the work done in displacing the charge by dl is

$$dW = -F \cdot dl = -QE \cdot dl.$$

The negative sign indicates that the work is being done by an external agent. Thus total work done, or the potential energy required, in moving Q from A to B, is

$$W = -Q\int_A^B E \cdot dl$$

Dividing W by Q in eqn. gives the potential energy per unit charge. This quantity, denoted by V_{AB}, is known as the potential difference between points A and B. Thus

$$V_{AB} = \frac{W}{Q} = -\int_A^B E \cdot dl$$

$$\text{where,} \quad E = \frac{Q}{4\pi\epsilon_0 r^2} a_r$$

$$\text{and} \quad V_{AB} = V_A - V_B$$

where, V_B and V_A are the potentials (or absolute potentials) at B and A, respectively.

→ The potential at any point is the potential difference between that point and a chosen point (or reference point) at which the potential is zero.

i.e., by assuming zero potential at infinite, the potential at a distance r from the point charge is the work done per unit charge.

$$\boxed{V = -\int_\infty^r E \cdot dl}$$

So, we define the potential, $V = -\int E \cdot dl$, it follows that

$$dV = -E \cdot dl = -E_x\,dx - E_y\,dy - E_z\,dz.$$

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But from calculus calculus of multi-variables, a total change in $V(x,y,z)$ is the sum of partial changes with respect to x, y, z variables

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} \cdot dz .$$

Comparing the two expressions for dV, we obtain

$$E_x = -\frac{\partial V}{\partial x} , \quad E_y = -\frac{\partial V}{\partial y} , \quad E_z = -\frac{\partial V}{\partial z}$$

Thus, $$E = E_x + E_y + E_z .$$

$$\boxed{E = -\nabla V}$$

~~Thus~~, That is, the electric field intensity is the gradient of V.

Poisson's and Laplace's Equation

→ Laplace's eq. give us method of finding potential function V when conducting materials in the form of planes curve, surface or lines are given and voltage on one is known with respect to some.

→ Laplace's eqn. is the special case of poisson's equation. To obtain poisson's eqn. from Gauss's law is very simple.

From Gauss's law,

$$\nabla \cdot D = \rho_v$$

but we know that $\bar{D} = \epsilon \vec{E}$ and $\vec{E} = -\nabla V$

$$\therefore \nabla \cdot (\epsilon \vec{E}) = \rho_v$$

$$\Rightarrow \nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$$

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$ for a homogeneous medium.

This is known as poisson's eq.

→ A special case of this eqn. occurs when $\rho_v = 0$ (i.e., for a charge-free region). Equation then becomes

$$\boxed{\nabla^2 V = 0}$$

Which is known as Laplace's equation.

Q. No.

Electric flux (Displacement)

→ In order to understand the concept of electric flux or displacement, we consider Faraday's experiment.

→ A sphere with charge +Q was placed within, but not touching, a larger hollow sphere.

→ The outer sphere was earthed momentarily and then the inner sphere was removed.

→ The charge remaining on the outer sphere was found to be of equal and opposite sign to the charge on the inner sphere (i.e., −Q).

→ Thus, we see that there was an electric displacement from the charge on the inner sphere through the medium to the outer sphere.

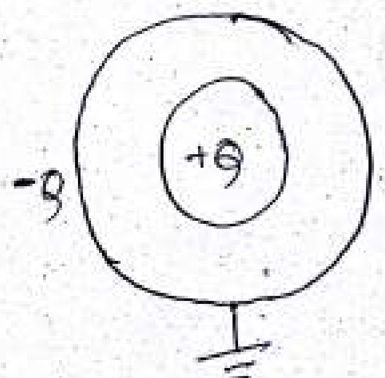

fig: Faraday's experiment.

→ This is termed as electric displacement or electric flux.

→ In SI units, this displacement is given as,

$$\psi = Q \quad \text{(Coulombs)}$$

Q. No.

In case of a point charge q, the outer sphere is assumed to have infinite radius.
The electric displacement per unit area of a charge q at the centre of a sphere of radius r is,

$$D = \frac{q}{4\pi r^2} \ C/m^2 \quad \text{—— (1)}$$

This is known as electric displacement density or flux density.

Now, for the same point charge, the electric field intensity at the centre of a sphere of radius r is,

$$E = \frac{q}{4\pi \varepsilon r^2} \quad \text{—— (2)}$$

from eqn (1) & (2), we get

$$\boxed{D = \varepsilon E}$$

In terms of flux density, $\vec{D}$, electric flux is defined as,

$$\boxed{\psi = \int_s D \cdot ds}$$

$q = \psi$

$D = \frac{\psi}{4\pi r^2}$

$\psi = D \cdot 4\pi r^2$

$= \oint_s D \cdot ds.$

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$$\vec{E} = \int_s \frac{\rho_s \, ds}{4\pi\epsilon_0 r^2} a_r \quad \text{(surface charge)}$$

3/ A volume charge is distributed throughout a sphere of radius a and centered at the origin with uniform density ρ_v c/m³. The total electric field is

$$E = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r^2} a_r \quad \text{(volume charge)}$$

Electric potential

work done = force × distance
= F dx

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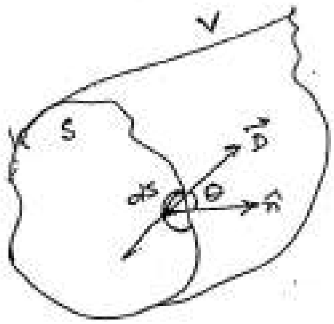

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If ρ_v is the charge density, then

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so, $\oint_s \vec{D}.\vec{ds} = \int_v \rho . dv$

applying Gauss's divergence theorem ~~both~~ lefthand sides of the equation.

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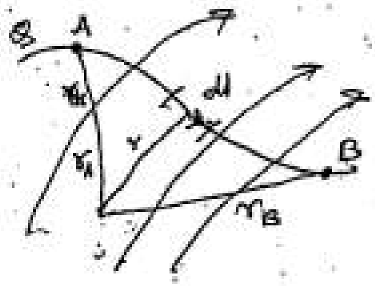

From coulomb's law, the force on Q is

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so that the work done in displacing the charge by dl is

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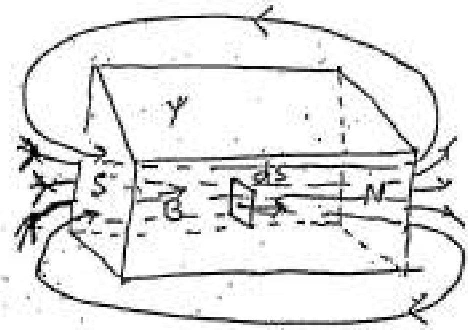
Magnetic flux density - Maxwell's Equation

→ The magnetic flux density B is similar to the electric flux density D.

→ As $D = \epsilon_0 E$ in free space, the magnetic flux density B is related to the magnetic field intensity H according to



$$\boxed{B = \mu_0 H}$$

where μ_0 is a constant known as the permeability of free space. The constant is in henrys per meter (H/m) and has the value of

$$\boxed{\mu_0 = 4\pi \times 10^{-7} \ H/m}$$

→ So, the magnetic flux (ψ) through a surface S is given by

$$\boxed{\psi = \int_S B \cdot ds}$$

Where the magnetic flux ψ is in Webers (Wb) and the magnetic flux density (B) is in Webers per square meter (Wb/m²) or teslas (T)

→ In an electrostatic field, the flux passing through a closed surface is the same as the charge enclosed; that is $\psi = \oint D \cdot ds = Q$.
Thus it is possible to have an isolated electric charge, which also reveals that electric flux lines are not necessarily closed.

→ Unlike electric flux lines, magnetic flux lines always close upon themselves. ~~That~~ This is because it is not posible to have isolated magnetic poles (or magnetic charges). Notice that each flux line is closed and has no beginning or end.

That is the total flux through a closed surface in a magnetic field must be zero.

$$\psi_m = \oint B \cdot ds = 0$$

Statement :- It states that the total ψ_m emerging through a closed surface is zero.

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This eqⁿ. is referred to as law of conservation of magnetic flux or Gauss's law for magnetostatic fields.

⇒ Although the magnetostatic field is not conservative, magnetic flux is conserved.

By applying the divergence theorem to eqⁿ., we obtain

$$\oint_s B \cdot ds = \int_v \nabla \cdot B \, dv = 0$$

$$\therefore \boxed{\nabla \cdot B = 0}$$

This eqⁿ. is the Maxwell's eqⁿ. to be derived. Equation shows that magnetostatic fields have no source or sinks.

(i) Energy store per unit volume or Electrostatic potential energy density:

$$u_e = \frac{1}{2}\epsilon E^2 = \frac{1}{2}\epsilon \vec{E} \cdot \vec{E} = \frac{1}{2}\vec{D} \cdot \vec{E}$$

∴ Total energy $= U_e = \iiint \frac{1}{2}\vec{D} \cdot \vec{E} \, dv$

$$\boxed{U_e = \int_v \frac{1}{2}\vec{D} \cdot \vec{E} \, dv.}$$

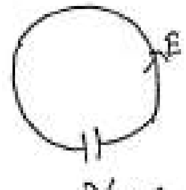
(ii) for Magnetostatic energy

$$\boxed{U_m = \int_v \frac{1}{2} \cdot \vec{B} \cdot \vec{H} \, dv}$$

Source of EMF

$$V_{emf} = \oint_l E \cdot dl$$

$$V_{MMF} = \oint_l H \cdot dl$$



we know that, electric field intensity around any closed path vanishes; i.e $\oint_l E \cdot dl = 0$.

From these eqs., it follows that a purely electrostatic field cannot cause a current to circulate in closed path (loop). In addition there must also exist a source of energy to maintain the steady current in closed loop.

The external source of energy may by non-electrical, such as battery, dc generator, solar cell, thermocouple.

In general, total electric field in a closed loop is

$$\bar{E} = \bar{E}_c + \bar{E}_e$$, where E_c is the static electric field due to charges, and E_e is the field generated by other causes (emf-producing field).

By defining the electromotive force (emf) in closed loop as

$$V_{emf} = \oint_l E_e \, dl = \oint_l E \cdot dl.$$

Similarly, mmf for magnetostatic field,

$$V_{mmf} = \oint_l H \cdot dl.$$

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Integral form of Faraday's law / Maxwell's 3rd eqⁿ.

Faraday's law state that the induced emf, V_{emf} (in volts), in any closed circuit is equal to the time rate of change of magnetic flux linkage by that circuit.

Mathematically express as.

$$V_{emf} = -\frac{d\psi}{dt}$$

where, V_{emf} → induced emf in the closed coil or circuit.

ψ → magnetic flux passing through it.

The negative sign shows that the induced voltage acts in such a way to oppose the flux producing it. This is known as Lenz's law.

Now, magnetic flux ψ passing through a closed circuit bounded by surface s can be written as

$$\psi = \oint_s \vec{B}\cdot ds$$

If the electric fields induced in space is $\vec{E}$, then the induced emf around the curve c,

$$V_{emf} = \oint_l \vec{E}\cdot dl$$

So, we get,

$$\oint_l \vec{E}\cdot dl = -\frac{d}{dt}\oint_s \vec{B}\cdot ds$$

$$\Rightarrow \boxed{V_{emf} = \oint_l \vec{E}\cdot dl = -\oint_s \frac{\partial \vec{B}}{\partial t}\cdot ds}$$

→ This is the Integral form of Faraday's law.

Applying the stoke's theorem left hand side of the eqn. ①

$$\oint_s (\nabla \times \vec{E})\, ds = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot ds \qquad \left[\oint_L \vec{A} \cdot dl = \oint_s (\nabla \times \vec{A})\, ds\right]$$

↳ stoke's theorem

$$\Rightarrow \oint_s \left(\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t}\right) ds = 0$$

$$\Rightarrow \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\Rightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ → Faraday's law of electromagnetic induction in differential form or point form or Maxwell's eqn.

Magnetic Scalar and vector potentials

→ We recall that some electrostatic field problems were simplified by relating the electric potential V & electric field intensity E ($E = -\nabla V$).

→ Similarly, we can define a potential associated with magnetostatic field B.

→ In fact, magnetic potential could be scalar V_m or vector A.

We know that, ampere's circuit law, for steady current,

$$\nabla \times \vec{H} = \vec{J}$$

if $\vec{J} = 0$ [For the region of free space]

then, $\nabla \times \vec{H} = 0$ --- ①

From vector property,

$$\nabla \times (\nabla V) = 0 \text{ --- } ②$$

Compare between ① & ② eqn., we get

$$\vec{H} = \nabla V$$

Just as $E = -\nabla V$, if V_m is the magnetic scalar potential

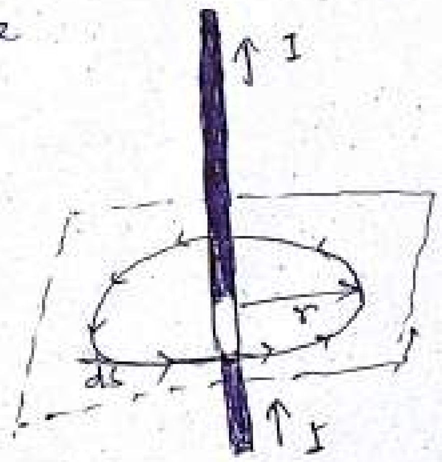
Ampere's circuit law

ACL state that the line integral of the magnetic field intensity ($\vec{H}$) around any closed path is equal to the direct current enclosed by the path.

$$\boxed{\oint_l H \cdot dl = I_{enc}.}$$

Proof: We consider an infinite straight conductor carrying a steady current I along the z-direction as shown in fig.

We consider a closed circular path of radius r enclosing the conductor.

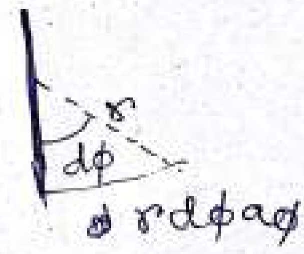


By the Biot-Savart law, the magnetic field intensity at any point on the circle is,

$$H = \frac{I}{2\pi r} \bar{a}_\phi$$

Now, the elemental length of the circle is

$$\vec{dl} = r\, d\phi\, \bar{a}_\phi$$



$$\therefore \bar{H} \cdot \bar{dl} = \left(\frac{I}{2\pi r}\bar{a}_\phi\right) \cdot r\, d\phi\, a_\phi$$

$$= \frac{I}{2\pi}\, d\phi$$

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$$\oint_l H \cdot dl = \frac{I}{2\pi} \oint_l d\phi = \frac{I}{2\pi} \cdot 2\pi$$
$$= I$$

$$\therefore \boxed{\oint_l H \cdot dl = I} \Rightarrow$$ ACL in integral form

now, applying stoke's theorem, we get

$$\oint_s (\nabla \times H) \cdot ds = I = \oint_s J \cdot ds$$

we get

$$\boxed{\nabla \times H = J} \Rightarrow$$ ACL in differential or point form.

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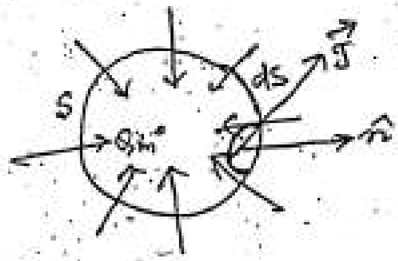
Continuity Equation

From the principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the surface of the volume.

Thus current I_{out} coming out of the closed surface is

$$I_{out} = \oint_S J \cdot ds = -\frac{dQ_{in}}{dt} \quad \text{--- (1)}$$



where Q_{in} is the total charge enclosed by the closed surface.

Let, ρ_v is the charge density per unit volume.

$$\text{so, } Q_{in} = \int_v \rho_v \, dv. \quad \text{--- (2)}$$

Invoking Gauss's divergence theorem, we write,

$$\oint_S J \cdot ds = \int_v \nabla \cdot J \, dv. \quad \text{--- (3)}$$

But,

$$-\frac{dQ_{in}}{dt} = -\frac{d}{dt}\int_v \rho_v \, dv = -\int \frac{\partial \rho_v}{\partial t} dv \quad \text{--- (4)}$$

Substituting eqn. (3) and (4) into eqn. (1) gives

$$\int_v \nabla \cdot J \, dv = -\int_v \frac{\partial \rho_v}{\partial t} dv.$$

$$\Rightarrow \boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$ ⇒ which is called continuity of current eqn. or first continuity eqn.

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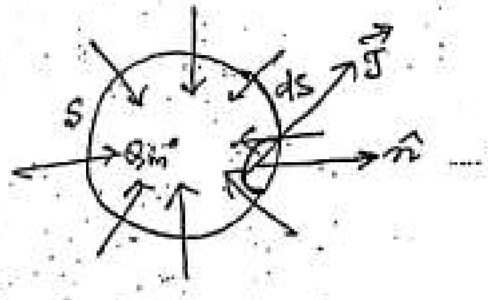

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$$\Rightarrow \boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$ $\Rightarrow$ which is called continuity of current eqn. or first continuity eqn.

Maxwell's postulate and displacement current

We shall now reconsider Maxwell's curl equation for magnetic fields (Ampere's circuit law) for time-varying conditions.

For static EM fields, we recall that

$$\nabla \times H = J \quad --- (1)$$

taking the divergence both sides of the eqn (1), we get,

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J$$

But the divergence of the curl of any vector field is identically zero.

Hence, $$\nabla \cdot (\nabla \times H) = 0 = \nabla \cdot J \quad --- (2)$$

We know that the continuity eqn for charge conservation is

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} = 0 \quad --- (3)$$

so, $$\frac{\partial \rho}{\partial t} = 0$$

i.e., ρ is constant.

Thus eqn (2) and (3) are obviously incompatible for time-varying conditions.

We must modify eqn (1) to agree with eqn.

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad --- (4)$$

To do this, we add a term to eqn (1) so that it becomes

$$\nabla \times H = J + J_d \quad ---- (5)$$

where J_d is to be determined and defined. Again, the divergence of the curl of any vector is zero. Hence,

$$\nabla \cdot (\nabla \times H) = 0 = \nabla \cdot J + \nabla \cdot J_d \quad --- (6)$$

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In order for eq^n. ⑥ to agree with eq^n. ④,

$$\nabla \cdot J_d = -\nabla \cdot J = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot D) = \nabla \cdot \frac{\partial D}{\partial t}$$

$$[\nabla \cdot D = \rho_v]$$

or,

$$\boxed{J_d = \frac{\partial D}{\partial t}} \quad \text{-----} ⑦$$

Substituting eq^n. ⑦ into eq^n. ⑤ results in

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

This is Maxwell's equation (based on Ampere's circuit law) for time-varying field.

~~The term $J_d = \frac{\partial D}{\partial t}$ is known as displacement current density, where D is displacement vector, and J is the conduction current density ($J = \sigma E$)~~

The term $J_d = \frac{\partial D}{\partial t}$ is known as displacement current density where D is displacement vector, i.e., the rate of change of electric field ($D = \epsilon_0 E$) with time is called displacement current density.

- Without the term J_d, the propagation of electromagnetic waves (e.g., radio or TV waves) would be impossible.
- At low frequency, J_d is usually neglected compared with J. However, at radio frequencies, the two terms are comparable.
- At the time of Maxwell, high freq. sources were not available and eq^n $\nabla \times H = J + J_d$ could not be verified experimentally.
- It was years later that Hertz succeeded in generating and detecting radio waves, thereby verifying eq^n $\nabla \times H = J + J_d$.

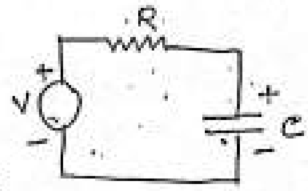
Based on the displacement current density, we defined the displacement current as:-

$$\boxed{I_d = \int J_d \cdot ds = \int \frac{\partial D}{\partial t} \cdot ds} \quad -- \; (8)$$

i.e., the current for which propagation of electromagnetic wave is possible, is known as displacement current.

~~Displace~~
Displacement → The act of displacing.

Let us consider, a circuit containing a parallel plate capacitor, a switch and a battery. When the switch is closed the current flows through the circuit.



And due to the applied voltage, the positive charges are accumulated in one plate of capacitor and negative charges are accumulated in another plate. This is the conduction current, we are familiar with it. But Maxwell's says that, the same amount of current will flow between the dielectric gap.

If σ be the charge density then $\sigma = \epsilon E$.

where E is applied electric field and

ϵ is the permittivity of the medium.

~~Rate of change of charge density is equal~~ to

$$\sigma = \epsilon E = D$$

$$\therefore \frac{\partial \sigma}{\partial t} = \frac{\partial D}{\partial t} = J_d$$

So, the rate of change of density is equal to rate of change of displacement current density.

For this displacement current is named so.

Maxwell's Equation for time varrying field

Differential form	Integral form	Name
i) $\nabla \cdot D = \rho$	$\oint_s D \cdot ds = \int_v \rho \, dv = Q$	Gauss's law of electrostatics
ii) $\nabla \cdot B = 0$	$\oint_s B \cdot ds = 0$	Gauss's law of magnetrostatics (non-existance of magnetic monopole)
iii) $\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_l E \cdot dl = -\frac{\partial}{\partial t} \int_s B \cdot ds$	faraday's law of electromagnetic induction
iv) $\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_l H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t} \right) ds$	Modified ampere's Circuit law

Statement 1

The total electric displacement through any closed surface enclosing a vollume is equal to the total charge within the volume.

Statement 2

The net magnetic flux emerging through any closed surface is zero. In otherwords, the magnetic flux lines do not originate and end anywhere, but are continuous.

Statement 3

21 The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

Statement 4

The magnetomotive force around a closed path is equal to the conduction current plus the time derivative of the electric displacement through any surface bounded by the path.

(ii) Maxwell's equations for free space

A free space has the following characteristics:

(a) volume charge density, $\rho = 0$

(b) conductivity, $\sigma = 0$

(c) Conduction current density, $J = \sigma E = 0$.

Therefore, Maxwell's equations for free space in differential and integral form will be

	Differential form	Integral form
i)	$\nabla \cdot D = 0$	$\oint_s D \cdot ds = 0$
ii)	$\nabla \cdot B = 0$	$\oint_s B \cdot ds = 0$
iii)	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_l E \cdot dl = -\int_s \frac{\partial B}{\partial t} \cdot ds$
iv)	$\nabla \times H = \frac{\partial D}{\partial t}$	$\oint_l H \cdot dl = \int_s \frac{\partial D}{\partial t} \cdot ds$

Maxwell's equations for static field

Static fields are those fields in which field vectors D, E, B and H do not vary with time (i.e., $\frac{\partial D}{\partial t} = \frac{\partial E}{\partial t} = \frac{\partial B}{\partial t} = \frac{\partial H}{\partial t} = 0$). Therefore, the Maxwell's equations in differential form and integral form will be

Differential form	integral form
i) $\nabla \cdot D = \rho$	$\oint_s D \cdot ds = \int_v \rho\, dv$
ii) $\nabla \cdot B = 0$	$\oint_s B \cdot ds = 0$
iii) $\nabla \times E = 0$	$\oint_l E \cdot dl = 0$
iv) $\nabla \times H = J$	$\oint_l H \cdot dl = \int_s J \cdot ds$

Maxwell's equations for sinusoidal time varying fields (Phasor Notation)

Any periodic variation can be analysed in terms of sinusoidal variation with fundamental and harmonic frequencies. These fields can be expressed by

$$E = E_0 \cos \omega t$$

or, $$E = E_0 \sin \omega t$$

where E_0 is the maximum field strength. The above expressions show that a sinusoidal time factor is attached to the field, but time factor may be suppressed by ~~with~~ use of phasor Notation.

The time varying fields $\tilde{E}(r,t)$ [~ is pronounced as tilde] can be expressed in terms of corresponding phasor quantity $\tilde{E}(r)$ as

$$\tilde{E}(r,t) = Re\left[E(r)e^{j\omega t}\right]$$

Differential form

i) In phasor Notation,

$$\tilde{D} = Re\left[D e^{j\omega t}\right]$$

~~for~~ from Maxwell's first equation

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot Re\left[D e^{j\omega t}\right] = \rho$$

$$\nabla \cdot Re\left[D e^{j\omega t}\right] - \rho = 0$$

$$\text{or, } \boxed{\nabla \cdot D = \rho}$$

ii) In phasor notation,

$$\tilde{B} = Re\left[B e^{j\omega t}\right]$$

From Maxwell's second equation,

$$\nabla \cdot \tilde{B} = 0$$

$$\nabla \cdot Re\left[B e^{j\omega t}\right] = 0$$

$$\text{or, } \boxed{\nabla \cdot B = 0}$$

iii) In phasor notation,

$$\tilde{E} = Re\left[E e^{j\omega t}\right]$$

$$\tilde{B} = Re\left[B e^{j\omega t}\right]$$

Now from Maxwell's third equation

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\text{or} \quad \nabla \times Re[E e^{j\omega t}] = -\frac{\partial}{\partial t}[Re[B e^{j\omega t}]$$

$$= -j\omega \, Re B e^{j\omega t}$$

$$\text{or,} \quad Re[(\nabla \times E + j\omega B) e^{j\omega t}] = 0$$

$$\text{or,} \quad \boxed{\nabla \times E = -j\omega B}$$

iv) In phasor notation,

$$\tilde{H} = Re[H e^{j\omega t}]$$

$$\tilde{D} = Re[D e^{j\omega t}]$$

$$\tilde{J} = Re[J e^{j\omega t}]$$

From Maxwell's fourth equation,

$$\nabla \times \tilde{H} = \tilde{J} + \frac{\partial \tilde{D}}{\partial t}$$

$$\nabla \times Re[H e^{j\omega t}] = Re[J e^{j\omega t}] + \frac{\partial}{\partial t} Re[D e^{j\omega t}]$$

$$= Re[J e^{j\omega t}] + j\omega \, Re[D e^{j\omega t}]$$

$$\text{or,} \quad Re[\nabla \times H - j\omega D - J] e^{j\omega t} = 0$$

$$\text{or,} \quad \boxed{\nabla \times H = J + j\omega D}$$

Differential form	Integral form
i) $\nabla \cdot D = \rho$	$\oint_s D \cdot ds = \int_v \rho \, dv$
ii) $\nabla \cdot B = 0$	$\oint_s B \cdot ds = 0$
iii) $\nabla \times E = -j\omega B$	$\oint_l E \cdot dl = -\int_s j\omega B \cdot ds$
iv) $\nabla \times H = J + j\omega D$	$\oint_l H \cdot dl = \int_s (J + j\omega D) \cdot ds$

Intrinsic Impedance

The wave impedance of an electromagnetic wave is the ratio of the transverse component of the E & H fields.

$$\eta = \frac{E}{H}$$

In terms of the parameters of an EM wave and the medium it travels trough, the wave impedance given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

for, free space,

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

unit → ohm.

Conductor

Conductors are the substrates which are used to carry or conduct electric charge from one point to other.

The conductors have the following properties under electrostatic conditions:

i/ Electric field inside the conductor is zero.
ii/ The conductivity of the conductor is infinite.
iii/ The charge density inside the conductor is zero.
iv/ The conductor including its surface is an equipotential region.

Proof :- Consider an isolated conductor, such as shown in fig, is placed in an electric field. That is, when external electric field E_e is applied, the positive free charges are pushed along the same direction as the applied field, while the negative free charges move in opposite direction. This charge migration takes place very quickly. The free charges accumulate on the surface of the conductor and form an induced surface charge and induced charge setup an internal induced field E_i, ... cancels the externally applied field E_e.

Thus, the flow stops; when $E_e = E_i$. As $E_e = E_i$ are equal and opposite, thus, net electric field inside the conductor is zero.

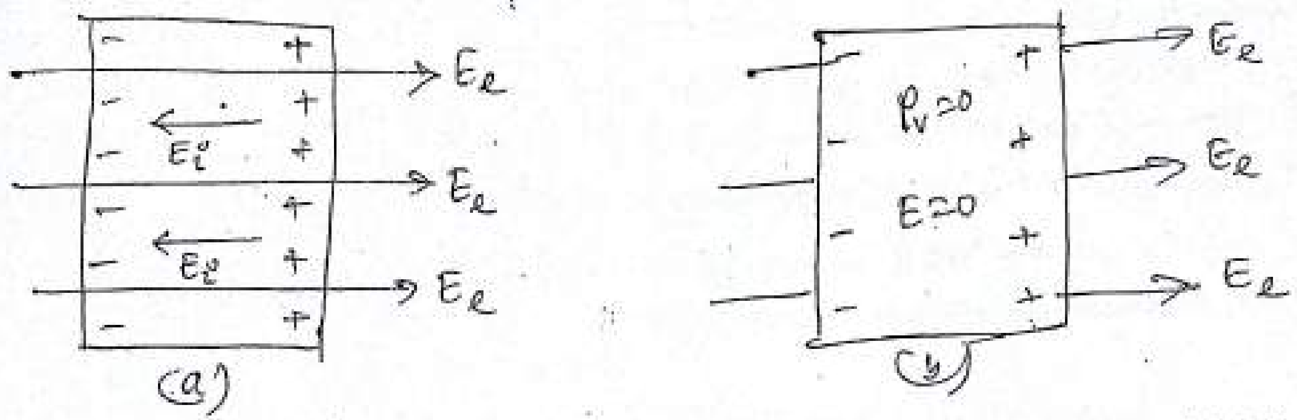


Fig:- An isolated conductor under influence of an applied field (b) A conductor has zero electric field under static condition.

Another way of looking at this is to consider Ohm's law, $J = \sigma E$. To maintain a finite current density J, in a perfect conductor ($\sigma \to \infty$), requires that the electric field inside the conductor $\sigma = \infty$, vanish.

In other words, $E \to 0$ because $\sigma \to \infty$ in a perfect conductor. According to Gauss's law, if $E = 0$ the charge density ρ_v must be zero.

$$\nabla \cdot D = \rho_v \,, \quad D = \epsilon E \,, \quad \nabla \cdot E = \frac{\rho_v}{\epsilon} \quad [\because E = 0]$$

$$\boxed{\rho_v = 0}$$

A conductor is called an equipotential body, implying that the potential is the same everywhere in the conductor. This is based on the fact that $E = -\nabla V = 0$.

$$\boxed{E = 0, \quad \rho_v = 0, \quad V_{ab} = 0 \quad \text{inside a conductor}}$$

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Boundary Conditions

We have considered the existance of the electric field in a homogeneous medium. If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called boundary conditions.

These conditions are helpful in determining the field on one side of the boundary if the field on the otherside is known. We shall consider the boundary conditions at an interface seperating

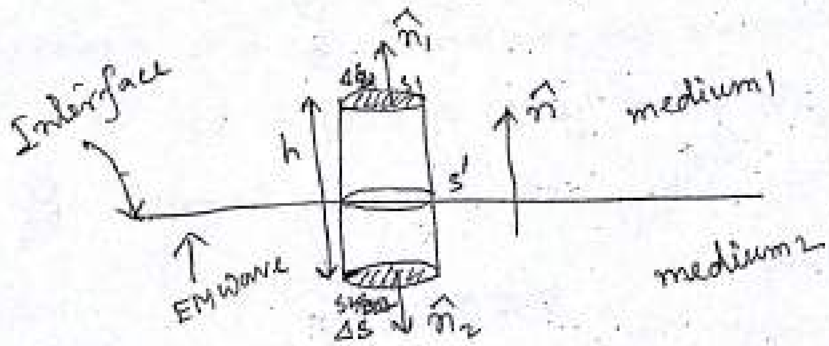
i/ Dielectric (ϵ_{r1}) and dielectric (ϵ_{r2})

ii/ conductor and dielectric

iii/ Conductor and free space.

Boundary conditions for Dielectric - Dielectric

Boundary Condition for $\vec{D}$



First, boundary condition for $\vec{D}$.

Now, Maxwell's first eq^n for electrostatic,

$$\nabla \cdot \vec{D} = \rho \quad \text{--- (1)}$$

Applying integral over the volume of the both sides of the eq^n (1), we obtain,

$$\int_v \nabla \cdot \vec{D} \, dv = \int_v \rho \, dv \quad \text{--- (2)}$$

Applying Gauss's divergence theorem of the left hand side of eqⁿ ②, we get

$$\oint_s \vec{D}\cdot ds = \int_v \rho\, dv = Q_{enc}.$$

$$\Rightarrow \int_{S_1} \vec{D_1}\cdot ds + \int_{S_2} \vec{D_2}\cdot ds + \int_{S'} \vec{D}\cdot ds = \int_v \rho\, dv. = Q_{enc}.$$

when $h \to 0$, $\int_{S'} \vec{D}\cdot ds = 0$

Allowing $h \to 0$ gives

$$\Delta Q = \rho\, \Delta s = D_1 \cdot \hat{n}_1\, \Delta s + D_2\, \hat{n}_2\, \Delta s$$

$$\hat{n}_1 = \hat{n}$$

$$\hat{n}_2 = -\hat{n}$$

$$\therefore\ D_1 \cdot \hat{n} - D_2 \cdot \hat{n} = \rho\, \frac{\Delta Q}{\Delta s} = \rho$$

$$\boxed{D_{1n} - D_{2n} = \rho}$$

where ρ is the free charge density placed deliberately at the boundary.

If no free chareges exist at the interface (i.e., charges are not deliberately placed there), $\rho_s = 0$ and eqⁿ becomes

$$\boxed{D_{1n} = D_{2n}}$$

(Thus the normal component of D is continuous across the interface; that is, D_n undergoes no change at the boundary.) Since $D = \epsilon E$, eqⁿ can be written as

$$\boxed{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

showing that the normal component of E is discontinuous at the boundary.

(iii) Boundary condition for conductor - dielectric

(In the case of conductor - dielectric boundary conditions. The conductor is assumed to be perfect (i.e., $\sigma \to \infty$ or $\rho_c \to 0$). Although such a conductor is not realizable for most practical purposes, we may regard conductors such as copper and silver as though they were perfect conductors.)

To determine the boundary conditions for a conductor - dielectric interface, we follow same procedure for dielectric - dielectric

⇒ we incorporate the fact that $E = 0$ inside the conductor, i.e.,

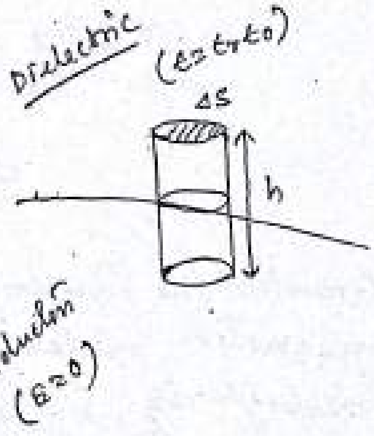
$$E_t = 0 \quad \text{for conductor medium.}$$

as $h \to 0$, we get

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S$$

because $D = \epsilon E = 0$ inside the conductor.

$$\text{so,} \quad D_n = \frac{\Delta Q}{\Delta S} = \rho_s.$$



$$\text{or,} \quad \boxed{D_n = \rho_s}$$

$$\text{or,} \quad \boxed{\epsilon_0 \epsilon_r E = \rho_s}$$

if, $\rho_s = 0$

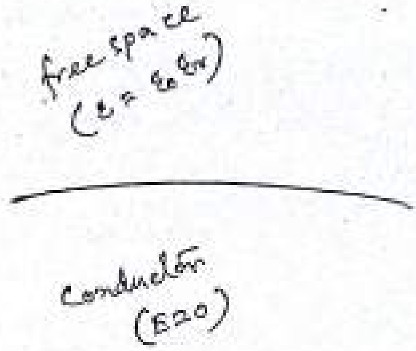
$$\boxed{\epsilon_0 \epsilon_r E = 0}$$

Boundary condition for conductor - free space

For free space may be regarded as a special dielectric for which $\varepsilon_r = 1$.

$$D_n = \varepsilon_r \varepsilon_0 E$$
$$= \varepsilon_0 E = 0 \quad [\because \varepsilon_r = 1].$$

and $\boxed{D_n = \varepsilon_0 E_n = \rho_s}$



Magnetic boundary condition

Boundary condition for B of Dielectric - Dielectric

Maxwell's eqn for magnetostatic,

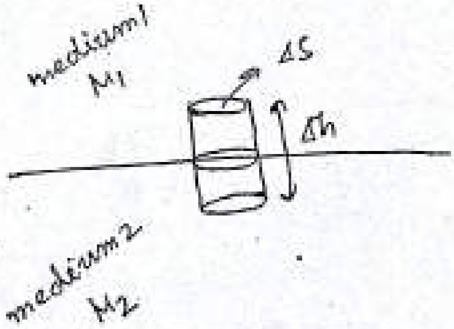
$$\nabla \cdot \vec{B} = 0 \quad \text{---} \ (1)$$

taking volume integral of the both sides, of the eqn.

$$\int_v \nabla \cdot \vec{B} \, dv = 0 \quad \text{---} \ (2)$$

applying divergence theorem of left hand side of the eqn, we obtain,

$$\oint_S \vec{B} \cdot ds = 0 \quad \text{---} \ (3)$$



Consider the boundary between two magnetic media 1 and 2, characterized, respectively, by μ_1 and μ_2 as shown in Fig. Applying eqn. (3) to the pillbox (Gaussian surface) of Fig. and allowing $\Delta h \to 0$, we obtain

$$B_{1n} \Delta S - B_{2n} \Delta S = 0$$

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Since eqⁿ shows that the normal component of B is continuous at the boundary.

$$B_{1n} = B_{2n}$$

$$\Rightarrow \boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$

It also shows that the normal component of H is discontinuous at the boundary.

- Boundary conditions for B for Conductor - Dielectric

For conductor, $H = 0$

$$B_{1n} = 0 \Rightarrow \mu_1 H_{1n} = 0$$

$$\mu_1 = \mu_r \mu_0$$

so, $\boxed{\mu_r \mu_0 H_{1n} = 0}$

- Conductor - Free space

For free space, $\mu_r = 1$

so, $\boxed{B_{1n} = 0}$

or $\boxed{\mu_0 H_{1n} = 0}$

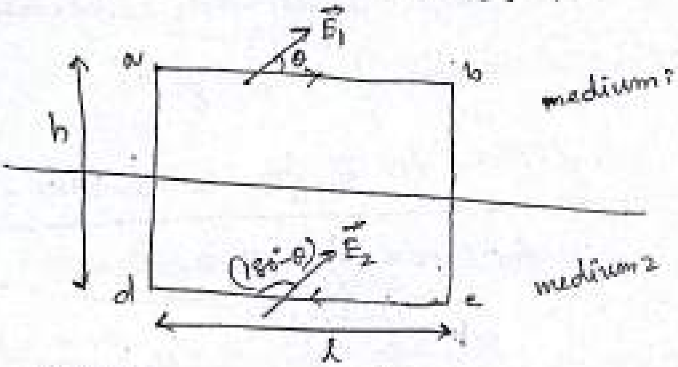
Boundary conditions for $\vec{E}$ for dielectric - dielectric

Maxwell's third eq^n is

$$\nabla \times E = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

taking surface integral of the both sides of the eq^n (1), we get.

$$\oint_s \nabla \times E \cdot ds = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot ds \quad \text{--- (2)}$$



applying ~~divergence~~ Stoke's theorem in left side of the eq^n (2), we get

$$\oint_l \vec{E} \cdot dl = -\oint_s \frac{\partial B}{\partial t} \cdot ds,$$

$$\Rightarrow \int_{ab} E_1 \, dl + \int_{cd} \vec{E}_2 \cdot dl + \int_{bc+ad} \vec{E} \cdot dl = -\oint_s \frac{\partial B}{\partial t} \cdot ds$$

When $h \to 0$, the $\int_{bc+ad} \vec{E} \cdot dl = 0$

again, we tell that, $h \to 0$ i.e. surface is zero, $\therefore ds = 0$

$$\therefore \oint_s \frac{\partial B}{\partial t} \cdot ds = 0$$

$$\therefore \lim_{h \to 0} \left[\int_{ab} \vec{E}_1 \cdot dl + \int_{cd} \vec{E}_2 \cdot dl \right] = 0.$$

$$\Rightarrow \vec{E}_1 \cdot \cos\theta \cdot l + \vec{E}_2 \cdot \cos(180° - \theta) \cdot l = 0$$

$$\Rightarrow \vec{E}_1 \cos\theta - \vec{E}_2 \cos\theta = 0$$

$$\boxed{\vec{E}_{1t} - \vec{E}_{2t} = 0}$$

and $$\boxed{\frac{\vec{D}_{1t}}{\varepsilon_1} = \frac{\vec{D}_{2t}}{\varepsilon_2}}$$

Thus the tangential component of E is continuous while D is discontinuous at the boundary.

~~Siti~~ Similar

Conductor - dielectric

$\varepsilon = \varepsilon_r \varepsilon_0$ and $E = 0$ for conductor.

$$\boxed{E_{1t} = 0}$$

and $$\boxed{\frac{D_{1t}}{\varepsilon_r \varepsilon_0} = 0}$$

conductor - free space

$\varepsilon = \varepsilon_r \varepsilon_0$ [$\varepsilon_r = 1$ for free space]

$= \varepsilon_0$

$$\boxed{E_{1t} = 0}$$

and $$\boxed{\frac{D_{1t}}{\varepsilon_0} = 0}$$

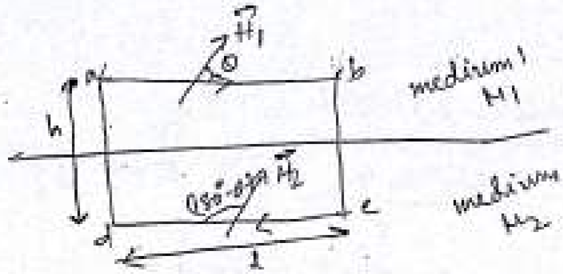
① Boundary condition for $\vec{H}$ for ~~[illegible]~~ dielectric - dielectric

Maxwell's eqn. magnetostatic,

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- ①}$$

taking the surface integral of the both side of the eqn ①, we get

$$\oint_s \nabla \times \vec{H} \cdot ds = \oint \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds \quad \text{--- ②}$$



applying stokes theorem in left hand side of the eqn ②, we obtain

$$\oint_l H \cdot dl = \oint_s \vec{J} \cdot ds + \oint_s \frac{\partial D}{\partial t} \cdot ds$$

$$\Rightarrow \int_{ab} \vec{H}_1 \cdot dl + \int_{cd} \vec{H}_2 \cdot dl + \int_{bc+ad} \vec{H} \cdot dl = \oint_s \vec{J} \cdot ds + \oint_s \frac{\partial \vec{D}}{\partial t} \cdot ds$$

when, $h \to 0$, then $\int_{bc+ad} \vec{H} \cdot dl = 0$.

and surface will be zero, $ds = 0$.

$$\Rightarrow \lim_{h \to 0} \left[\int_{ab} \vec{H}_1 \cdot dl + \int_{cd} \vec{H}_2 \cdot dl \right] = J_L \cdot l \quad \text{[linear current density]}$$

$$\Rightarrow \vec{H}_{1t} \cos\theta \, l + \vec{H}_{2t} \cos(180° - \theta) \cdot l = J_L \cdot l$$

$$\Rightarrow \boxed{\vec{H}_{1t} - \vec{H}_{2t} = J_L}$$

If the boundary is free of current or the media are not conductors, $J_L = 0$ and eqn. becomes

$$\boxed{\vec{H}_{1t} = \vec{H}_{2t}}$$

or, $$\boxed{\frac{\vec{B}_{1t}}{\mu_1} = \frac{\vec{B}_{2t}}{\mu_2}}$$

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A

Thus the tangential component of H is continuous while that of B is discontinuous at the boundary.

conductor - dielectric for H

For conductor, $H = 0$

$\mu = \mu_r \mu_0$

$$\boxed{\vec{H_{1t}} = 0}$$

and $$\boxed{\frac{\vec{B_{1t}}}{\mu_r \mu_0} = 0}$$

conductor - free space. for H

$\mu_r = 1$ for free space.

$$\boxed{\vec{H_{1t}} = 0}$$

and $$\boxed{\frac{\vec{B_{1t}}}{\mu_0} = 0}$$

(1) Explain the Gauss's law in electrostatic and derive Maxwell's eqn $\nabla \cdot D = \rho$

or, state and explain Guss's law for electro-static.

(2) Why is the divergence of magnetic flux density B always zero?

(3) State faraday's law. from the fundamental principle of faraday's law, establish the relation $\nabla \times E = -\frac{\partial B}{\partial t}$.

(4) State and explain ampere's circuit law.

or, prove that $\nabla \times H = J$.

(5) Starting from ampere's circuit law, establish the relation $\nabla \times H = J + \frac{\partial D}{\partial t}$

or
Explain the inconsistancy present in the ampere's circuit law. How the law is modified by Maxwell's, explain.

and what is displacement current and why so called?

(6) Write the mathematical expression for the law of conservation of charge. Hence obtain the equation of continuity.

or write a short note on continuity eqn.

(7) Write down the Maxwell's eq^n. for electro-static and magnetrostatics.
or write down the differential (Point) and integral form of maxwell's four eq^{n}s in time varying EM fields.

(8) Establish the boundary conditions for electric and magnetic field intensities and the interface between two dielectric media.
Explain ~~to~~ how these conditions will be modified, if one of the media is a perfect conductor.
or Write a short note on boundary conditions for electric and magnetic fields.

Problem

Q. No.

1. Electric field components of a propagating electromagnetic wave in free space are given as

$E_x = E_z = 0$ and $E_y = E_0 \cos(\omega t - \beta z)$.

Determine the field components of the magnetic field.

$H_x = \frac{\beta E_0}{\mu_0 \omega} \cos(\omega t - \beta z)\, a_x$ A/m.

2. If $x<0$ defines region 1 and $x>0$ defines region 2, then find the electric field intensity in region 2 ($\varepsilon_{r_2} = 5$), if E in region 1 ($\varepsilon_{r_1} = 1$) is $\vec{E}_1 = (4\hat{u}_x + 1.5\hat{u}_y - 2\hat{u}_z)$ V/m.

$E_2 = 0.8\hat{u}_x + 1.5\hat{u}_y - 2\hat{u}_z$ V/m.

3. Two homogeneous dielectric regions 1 ($\rho \le 4$ cm) and 2 ($\rho \ge 4$ cm) have dielectric constants 3.5 and 1.5 respectively. If $D_2 = 12a_\rho - 6a_\phi + 9a_z$ nC/m², calculate i) E_1 and D_1 (ii) P_2 and ρ_{pv2} (iii) the energy density for each region. $D_1 = 12a_\rho - 14a_\phi + 21a_z$ nC/m²

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4. A boundary exists at $z=0$ between two dielectrics $\varepsilon_{r_1} = 2.5$ region $z<0$, and $\varepsilon_{r_2} = 4$ region $z>0$. The field in region of ε_{r_1} is $E_1 = 30a_x + 50a_y + 70a_z$ V/m, find (a) normal component of E_1 (b) tangential component of E_1 ~~(c) the angle $\alpha_1 \le 90°$ between E_1 and normal to the surface~~. (d) normal component of D_2 (e) tangential component of D_2 (f) D_2.

$E_{n1} = 70$ V/m, $E_{t1} = 58.3$ V/m, $D_{n2} = 1.549 \times 10^{-9}$ C/m², $D_{t2} = 2.833 \times 10^{-9}$ C/m²

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(5) In free space, electric field strength, $E = E_0 \sin(\omega t - \beta z)\, a_y$ V/m. Find magnetic flux density B.

$B = -\frac{\beta}{\omega} E_0 \sin(\omega t - \beta z)\, a_x$ Wb/m²

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